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THE UNIVERSITY OF ALBERTA

GRADE SIX STUDENTS' METHODS OF ESTIMATING ANSWERS TO COMPUTATIONAL EXERCISES

BY

© CAROL MARIE HAUK

A THESIS

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND RESEARCH IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF EDUCATION

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ABSTRACT

There is wide agreement that estimation skills are important and should be taught as an integral part of the mathematics program. Few studies have been conducted, however, to determine how children estimate and why they experience difficulty with this topic. The major purpose of the present study was to determine what strategies grade six students used when they estimated answers to computational exercises in addition, subtraction, multiplication and division of whole numbers.

Twenty-two grade six students from the Edmonton Catholic System participated in the study. There were ten girls and twelve boys from a class grouped heterogeneously with respect to ability.

A two-part interview was conducted with each student on an individual basis. The first part required that the student orally estimate answers to computational exercises in addition, subtraction, multiplication and division. The second part required the student to mentally round numbers and then mentally compute with those rounded numbers. The interviews were tape-recorded to facilitate subsequent analysis.

Results showed that the students were more accurate when estimating answers to addition and subtraction exercises than multiplication and division exercises. The majority of the estimates could generally be identified with use of rounding procedures or mental use of pencil and paper algorithms. No particular procedure was

associated with either the "accurate" or the "inaccurate" estimates. The ability to round numbers and compute with those rounded numbers appeared to be neither necessary nor sufficient to make accurate estimates.

Major recommendations were that children should have experience in making estimates in a variety of practical settings and in deciding upon reasonable accuracy for the answer.

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CHAPTER I

INTRODUCTION AND STATEMENT OF THE PROBLEM

Purpose of the Study

The purpose of the study was to determine what strategies grade six students used as they estimated answers to computational exercises in addition, subtraction, multiplication and division of whole numbers. Verbal interaction with the students while making the estimates was the method used to obtain knowledge of the thought processes used by the students.

It was hoped that the data obtained in the study would provide teachers with insight into the strategies children use when they estimate so this knowledge could be incorporated into teaching methodology.

Rationale for Estimation

Reporting on the National Assessment of Educational Progress (N.A.E.P.), Carpenter et al (1976) claimed that in view of its importance, estimation may be one of the most neglected skills in the mathematics curriculum. Estimation is a broad topic, used in reference to number, measurement, and computation. This study dealt with estimation only as it applied to computation.

There is wide agreement that developing estimation skills should be an objective of the mathematics curriculum. Sauble (1955) gave both a social need and a mathematical need. The former arises from the frequent use of mental arithmetic in daily life where estimates can serve as a check on the exact answer, or may be

adequate in themselves. There is a mathematical need because mental computation and estimation not only provide opportunities for the utilization of acquired meanings, but help to further understandings of basic number principles and relationships.

Payne and Seber (1959) claimed that a design for learning which includes planned experiences in estimating and making sensible approximations is necessary, as children are highly unlikely to achieve these skills by themselves. Holmes (1975) agreed in saying that estimation is an important skill for real living and class time devoted to it is well spent.

Edwards, Nichols and Sharpe (1972), reporting for the Committee on Basic Mathematical Competency and Skills, proposed guidelines which would define minimum competence in mathematics. Estimation skills were included as necessary for judging the reasonableness of computational answers, and for making intelligent consumer decisions.

Bell (1974) proposed a specification of what is "really" needed for minimum mathematical literacy and competency in the mathematics curriculum. He called for confident, ready and informed use of estimates and approximations.

Trafton and Suydam (1975) presented a position paper on computational skills which reflected the thinking of the Arithmetic Teacher editorial panel. The second of ten points stated: "All children need proficiency in recalling basic number facts, in using standard algorithms with reasonable speed and accuracy, and in estimating results and performing mental calculations, as well as an understanding of computational procedures (p. 530)."

More recently, the wide availability of the hand-held calculator has had implications for the entire mathematics curriculum. Opponents of calculator use predict the decline and eventual loss of mental arithmetic reasoning. Advocates, on the other hand, while cautious about elementary school use of the calculator, claim it can and should be used to advantage. Many educators, including Bruni (1976), Ockenga (1976), Gibb (1975), and Immerzeel (1976) have pointed out the necessity for sharpening estimation skills when using the calculator. Bell (1976) reported that poor judgment of significant figures is revealed by calculator use. Calculator errors tend to be large and ability to estimate would help prevent students from accepting erroneous answers. Bell suggested that estimation skills, which are important in any case, must be learned.

Rationale and Need for the Study

There is little or no argument that estimation skills are important for modern living. Results from the N.A.E.P. (1976), however, gave evidence that present treatment is insufficient to build any real skill in estimating or to encourage students to use estimation where it is appropriate.

There have been few studies conducted to identify the cause of children's difficulties with estimation. One reason for this is that estimation is a process and is difficult to measure. Trafton (1978) said that there is not much known about how children's thinking develops in this area, nor how they can be helped to learn estimation skills competently and confidently. Investigating

children's estimation techniques, Sauble (1955) found that they do not always employ standardized prescribed thought patterns, but use ingenuity and resourcefulness.

The N.A.E.P. study (1976) hypothesized that one source of difficulty in making estimates is a lack of the prerequisite skill of rounding numbers and operating with multiples of ten, one hundred, one thousand and so on. Further indications were that estimation requires "quantitative intuition" which includes the ability to decide if an answer makes sense, and a feeling for the quantities represented by large numbers. There is no well-defined algorithm for estimation; any such strategy would depend on the acceptable criteria for a reasonable estimate.

N.A.E.P. proposed, ". . . it is doubtful that the ability to estimate can be validly assessed if only the final result of the calculation is considered. To obtain a valid measure of a student's ability to estimate, it is probably necessary to observe that student estimating (p. 299)."

Scope of the Study

The study was designed to help answer the following questions:

1) How accurately do children estimate answers to computational exercises in addition, subtraction, multiplication and division?

2) What identifiable strategies do children use when estimating?

3) Are there similarities in the strategies used to obtain "accurate" estimates?

4) Are there similarities in the strategies used to obtain "inaccurate" estimates?

5) Is there a relationship between the ability to make accurate estimates and the ability to round numbers and compute with those rounded numbers?

Contribution of the Study

The current paucity of knowledge regarding students' understanding and use of estimation prompted this study which is exploratory and descriptive in nature. The study was designed to yield information about the strategies children use when they estimate. Knowledge of strategies used and difficulties encountered by students would have implications for teaching methodology. It was also hoped that the results would provide some direction for future research in this area.

CHAPTER II

REVIEW OF THE LITERATURE

Estimation and Mental Arithmetic

Estimation as a topic cannot be completely separated from the larger field of mental arithmetic. Mental arithmetic has had an interesting history which provides a background for the present study.

History of mental arithmetic

Mental arithmetic is a typical example of a mathematics curriculum concern whose popularity has been cyclical in nature. Historically, as pointed out by Wolf (1960), it long preceeded written computation. Following the advent of written numerals, numerical systems were perpetuated by pencil and paper for the purposes of educating the young and satisfying the needs of society. Resulting materials tended to promote routine pencil and paper procedures, which when emphasized, became a guide to arithmetic instruction rather than an aid. This written approach to teaching arithmetic relegated oral computation to a minor role.

According to Smith (1913), a change was initiated in the early nineteenth century, largely due to the influence of Pestalozzi in Europe and Warren Colburn in the United States. They protested the written method of arithmetic, claiming it was too slow and promoted intellectual sluggishness and lack of reasoning. Also prominent at this time was the theory of faculty psychology which advocated mental

discipline through exercising the mind with mental calculations. Thus, oral arithmetic flourished and written arithmetic assumed the minor role.

In the early twentieth century, the theory of faculty psychology was discredited. In the large reaction against the doctrine of mental discipline, mental arithmetic once again fell into great disfavor. Hall (1954) regrets this erroneous association by writing, "It is unfortunate that mental arithmetic, in such questionable company as mental discipline, was also discredited (p. 349)."

The result was that, from about 1910 to 1940, mental arithmetic virtually disappeared from the curriculum. By the 1940's, educators once more began to reexamine the place of mental arithmetic in the curriculum. This was perhaps due to the social utility theory of curriculum prevalent at that time. Mental arithmetic for everyday life situations was recognized as being both necessary and different from pencil and paper methods.

Until this time, arithmetic instruction tended to extremes of total emphasis on, or total lack of, mental procedures. According to Hall (1954), the return was characterized by a totally different point of view. Purposeless drill on rapid calculation was replaced by practical situations which required either exact or approximate answers, methods which led to increased understanding of the decimal system of numeration and number relations, and enrichment exercises for motivation. The past tendency to adopt extreme positions was seen as impractical. Putting it in its proper perspective,

Thorndike (1922) said that problems solved with or without pencil and paper had varying degrees of merit, according to the particular arithmetic exercise, pupil, and problem context. A similar thought was voiced by Suzzallo (1911) who said that the use of pencil and paper or mental procedures should be a matter of situational judgment rather than an arbitrary decision which is often uneconomical or inefficient.

Definition of mental arithmetic

Another factor which has influenced the popularity of mental arithmetic is definition of the term itself. Hall (1954) found that many different terms and phrases are understood and used in reference to mental arithmetic by textbooks, authorities, and teachers. He discovered that in eight series of fifth and sixth grade textbooks, eleven different terms or phrases were used in reference to solving problems mentally. Brown (1950) surveyed teachers on their opinion as to whether or not mental arithmetic should be included in a cultural general mathematics course. He found that their answers depended upon their notion of mental arithmetic. Those in favor interpreted it as a topic which implied estimation or mental shortcuts; those not in favor saw it as using standard written procedures, but doing them mentally.

The terms mental arithmetic and oral arithmetic are used most frequently, and are sometimes used synonomously. Some authorities (Hall, 1954) claimed that all arithmetic is mental; even when each step is written down, the arithmetic is done in the mind.

One such contender was Thompson (1917), who saw mental arithmetic as calculations performed in the mind without recording on paper any intermediate results, whether the final answer is written down, said aloud, or simply thought. Similarly, the origin of the problem may be spoken, written or raised in the mind by a practical situation. This view was adopted by Schall (1973) and Hall (1954) who also included quick estimations which may or may not be verified in writing. Hall recommended exclusive use of the term mental arithmetic because it is used more frequently than oral arithmetic and because the latter term often limits the problem to one which arises, is solved, or answered orally.

Flournoy (1959) defined mental arithmetic as "the interpreting or solving of quantitative situations without the aid of paper and pencil (p. 133)." She defined short-cut method as "a method which differs from the regular paper and pencil method. Sometimes it may not actually be shorter but it may be an easier way of thinking when not using paper and pencil (p. 137)." Wolf (1960) used the term oral arithmetic which he interpreted as non-written figuring. These definitions would not seem to limit mental arithmetic to refer only to exact calculations, but could encompass situations in which estimates are made of numerousness, measurements or answers to calculations.

Need for mental arithmetic

To justify the inclusion of mental arithmetic in the curriculum, it must be recognized as differing from pencil and paper methods, and a need for such procedures must be established. While the topic has

not been extensively investigated, a review of existing journal articles and research reports offers much support.

A major proponent of the teaching of mental arithmetic, Frances Flournoy (1957) asserted that it is a matter of common experience and observation that mental arithmetic is often required to solve or interpret quantitative situations or produce quick solutions without the use of pencil and paper. Children must be given the opportunity to mentally solve problems involving simple computation, make approximations, and interpret quantitative data, terms and statements. In a study with intermediate grade children, trained to mentally solve exercises and word problems, Flournoy (1954) claimed they made significant gains in these skills. Their written arithmetic skills were not hindered, but rather seemed to be aided by the experience. Supported by her research, she concluded that it is doubtful that good arithmetic teaching which emphasizes only written skills, equips pupils to handle daily mental arithmetic situations. This view reaffirmed a similar one made by Smith (1913) more than forty years earlier.

Wandt and Brown (1957) investigated the non-occupational use of mathematics. In a twenty-four hour period, college students kept a record of their uses of computation. Mental computation accounted for 75% of their uses and written computation for 25%. Of the uses recorded by children in grades three to six, 71.6% were non-pencil-and-paper. It was felt that the children may have overlooked many uses and that they would make even greater use of

mental arithmetic if they felt confident in doing so. The reading of tables, graphs, and scales found in reference materials and newspapers involves mental arithmetic, but such uses were not reported, indicating that they were overlooked.

Kramer (1970) said mental computation is one of the skills which primary teachers must develop in young children, as it will be of substantial assistance to them throughout life. His claim for this emphasis is that about 75% of all non-occupational uses of arithmetic by adults is mental. Beberman (1965) observed that mental arithmetic helps children become independent of memorized techniques. Children who follow strict algorithms with pencil and paper are at a great disadvantage when following blackboard or textbook presentations. Mental arithmetic encourages children to discover computational shortcuts which provide deeper insight into the number system.

Payne (1966) found that grade five pupils who were taught mental computation using a specified time allotment within the arithmetic curriculum and a step-by-step planned sequence of material performed better than those using an intermittent textbook presentation, as measured by a mental computation test. As standard achievement test scores revealed no significant differences between the two groups, he concluded that the time taken from regular class instruction had no adverse effect on the experimental group.

In a similar experiment, Grumbling (1970) found that grade four pupils significantly improved their mental computation skills following a series of prepared mental computation lessons, without

suffering a loss in arithmetic achievement. The control group, following the regular arithmetic program, also made significant gains in mental computation ability, but the experimental group made significant gains in mental problem-solving ability as compared to the control group.

Rea and French (1972) assessed the merits of oral instruction in mental computation versus pencil and paper enrichment activities for grade six. Both treatments, as measured by standardized mathematics achievement test scores, resulted in a significant growth over a small number of instructional sessions.

Wolf (1960) conducted a study to determine the best materials and procedures for oral instruction in the four fundamental processes as well as addition and subtraction of fractions. The lessons were geared to supplement grade four and five arithmetic programs. Experimental groups received either film presentations or printed materials. He found that both experimental methods were effective in both motivating and teaching oral arithmetic processes.

Similarly, Schall (1973) compared the effects of short, frequent mental arithmetic lessons in grade five, which were presented in either visual, oral, or both visual and oral modes. The pupils grew in their mental arithmetic skills regardless of the mode of presentation, all of which were enjoyed. There was no loss in the paper and pencil skills of the experimental groups, though regular class time was used for the lessons. In fact, these groups showed gains in the problem-solving retention test, indicating that they were

better able to transfer skills and concepts than the control groups who received either no treatment or placebo treatment.

Hall (1947) investigated mental problem solving with grade six pupils. He found there are considerable individual differences in ability to solve verbal problems without pencil and paper. There was a positive correlation with I.Q., but no significant difference between male and female students. Generally, addition problems were the easiest, followed by subtraction, division and multiplication, in that order. Similarly, one-step problems were easier than two or three step problems.

Spencer and Brydegaard (1952) claimed that oral-verbal problems occur more often in life experiences than do written problems. In school, these problems should center around children's activities and consumer type problems. Spitzer (1954, 1967) said that if he were limited to offering one suggestion for improving the problem-solving ability of upper-grade pupils, it would have to be oral problem solving. He claimed it is successful because it puts into practice essential problem-solving procedures. These he listed as: 1) undivided attention to the problem, 2) sizing up or analysing the problem situation, 3) use of significant or simple numbers, 4) better picturing of the problem-solving process through forced use of simple numbers and processes, 5) time for reflection and evaluation, and 6) better understanding through various solution methods.

Petty (1956) conducted two parallel experiments to study the effectiveness of a non-pencil and paper method of solving verbal

arithmetic problems in grade six. The experimental groups did not use pencil and paper and had no access to the printed problem during the alotted solution time. Control groups used pencil and paper and had continuous presentation of the printed problem during both reading and solution time. Criterion tests included pencil and paper problems, non-pencil and paper problems, orally stated problems, and a speed test. There were no significant differences on the overall mean gains or performance changes between the experimental and control groups, but gains made favored the group whose practice method was similar to the criterion method. This trend indicated that maximum improvement in ability to solve verbal problems by either method could best be accomplished by practice in the respective method. While the control group scored higher on the practice exercises than did the experimental groups, their advantage declined as the experimental group had more practice with non-pencil and paper methods. Since there is a need in life to solve problems mentally, practice in such procedures is essential.

Mental arithmetic, then, whether it refers to solving problems or performing computations without pencil and paper, has a definite place in the curriculum. Mental arithmetic situations occur frequently in life for both children and adults. The processes involved differ from pencil and paper procedures, and require specific instruction if they are to be used both effectively and efficiently.

Research on Estimation of Computation

The literature on estimation of computation is sparse and the studies done vary in nature. They include investigation of the effect that teaching estimation has on computational skills, the relative merits of planned or incidental instruction in estimation and attempts to correlate the ability to estimate with other skills or attributes.

Mazzei (1959) compared the growth in arithmetic achievement of grade nine and ten pupils who estimated answers to problems before solving them, with those who did not. He found that the lower one-third of the experimental classes achieved greater gains than the upper third in three-quarters of the comparisons made. This indicated that the lower students can increase achievement skills by first estimating answers. He was certain that arithmetic instruction would become more meaningful and students would show increased understanding if emphasis were placed upon a thoughtful analysis of the relationships between numbers rather than upon rule-of-thumb procedures.

Nelson (1966) compared the effect of planned teaching of estimation on arithmetic achievement with the effect of incidental estimation exercises in a regular textbook in the fourth and sixth grades. In grade six, the experimental group significantly surpassed the control group on a test of arithmetic concepts and applications. In grade four, the control group surpassed the experimental group on a test of computation. The experimenter felt that this was perhaps due to the fact that the time necessary to estimate each answer prevented them from completing as many examples. The children given instruction and practice in applying estimation skills were more competent in using this procedure than those not receiving the

instruction. This indicated that the ability to estimate is more related to the ability to apply procedures than the ability to compute. Also, children with higher I.Q.'s tended to be better estimators. Nelson concluded that it is difficult to successfully integrate such a program in one year, as change in behavior is related to previous habits and teaching.

Paull (1971) analysed grade eleven student's ability to estimate length, area, and answers to numerical computations; and to solve problems by trial and error. He found that the subjects were not consistent across the tasks in their ability to estimate answers to problems, but this ability was significantly correlated with mathematics and verbal ability. He also found a positive correlation between the ability to estimate answers to numerical computation and the ability to solve problems by trial and error. The ability to estimate length was not correlated with mathematical or verbal ability nor was the ability to estimate area and length correlated with the ability to compute rapidly. Estimation abilities were not related to the tendency to categorize quantities in broad, medium or narrow band-widths, nor were there significant differences between males and females.

Mental arithmetic has had an interesting history. Acceptance or rejection of mental procedures as part of the mathematics curriculum has been inconsistent. This inconsistency was in part due to the difficulty of defining mental arithmetic. Those educators who have seen mental arithmetic as procedures different from pencil and paper

algorithms, have promoted its practice as part of regular mathematics instruction.

Estimation is an important, but often neglected, facet of mental arithmetic. Previous studies concerning estimation have been statistical in nature, mainly attempting to link the ability to estimate with other measurable mental abilities. The present study is intended to be descriptive of the strategies used to estimate and how the strategies are related to the accuracy of the estimates.

CHAPTER III

DESIGN OF THE STUDY

The main purpose of the study was to determine what strategies grade six students used as they estimated answers to computational exercises in addition, subtraction, multiplication and division of whole numbers. The specific purposes of the study were listed in Chapter I. The present chapter includes definition of terms, description of the sample and instrument used, method for data collection, assumptions and limitations, and an outline of the data analysis plan.

Definition of Terms

The following definitions are operational in nature as used in the study:

Estimation:	the mental process of determining an approximate answer to a computational exercise.
Estimate:	the final answer determined through the process of estimation.
Proper order of magnitude:	within one thousand, one hundred, ten or one of the exact answer when the largest place value of the exact answer is thousands, hundreds, tens or one respectively.
Accurate estimate:	within the proper order of magnitude.
Inaccurate estimate:	outside the proper order of magnitude.

Description of Sample

The sample was comprised of twenty-two grade six students from a school in the Edmonton Catholic System. The area was described by the administration as middle-class. Twelve boys and ten girls participated in the study. The class was heterogeneously grouped with respect to ability.

Table 1 shows existing data which were available for the sample.

The students were listed in descending order of age as of April 1, 1978. This statistic was the only one which was available for the entire sample. The average age was eleven years and six months; and the ages ranged from eleven years and two months to twelve years and zero months.

As the entrance age for grade one students in the Edmonton Separate System ranges from five years and six months to six years and six months, the ages of grade six students as of April 1 would be expected to range from eleven years and one month to twelve years and one month. The ages of all students in this sample fall within the expected range.

Lorge-Thorndike I.Q. scores of twenty students were available. Of these twenty, the average score was 105 and the scores ranged from 72 to 128. As the average score for the Edmonton Separate System is 104, the students in this sample scored slightly above the norm for the system.

Table 1

Ages in years and months, I.Q. scores, and grade equivalent scores for concepts and problems on Canadian Test of Basic Skills, for sample used.

Student	Age	I.Q.	Concepts	Problems
1	12- 0	105	6.9	6.9
2	11-11	111	6.3	5.9
3	11-11	93	5.5	4.3
4	11-11	122	6.8	6.4
5	11-10	116	5.8	6.4
6	11- 9	109	5.9	5.7
7	11- 7	106	5.4	5.8
8	11- 7	100	5.8	4.9
9	11- 7	97	-	-
10	11- 6	115	6.8	7.3
11	11- 6	-	5.9	5.7
12	11- 5	128	7.9	7.3
13	11- 5	116	6.5	6.4
14	11- 5	-	4.4	3.2
15	11- 4	110	5.9	5.1
16	11- 4	113	-	-
17	11- 3	104	3.5	3.6
18	11- 3	72	3.1	4.6
19	11- 2	97	5.4	5.8
20	11- 2	118	6.7	6.2
21	11- 2	84	4.2	4.9
22	11- 2	91	4.2	4.3
Average (based on scores available)	11- 6	105	5.6	5.5

Scores from the Canadian Test of Basic Skills administered in April, 1977 were available for twenty students. Scores for both the concepts and problems subtests are reported in grade equivalents. The average grade equivalent for concepts was 5.6 and the scores ranged from 3.1 to 7.9. The average grade equivalent for the problems was 5.5 and the scores ranged from 3.6 to 7.3. As the expected average for both these scores for students in the Edmonton Separate System would have been 5.7, the scores of the students in this sample were slightly lower than expected.

Thus, when considering age, I.Q. and mathematics achievement, this class is close to the norms for sixth grade classes in the Edmonton Catholic System.

Grade six pupils were chosen because children at this level are expected to be proficient at doing the four basic operations with whole numbers. It was also felt that children at this age would be better able to explain their strategies than would younger children capable of the same exercises.

Instruments and their Characteristics

A two-part interview was conducted with each child on an individual basis. The first part required that the student orally estimate answers to computational exercises in addition, subtraction, multiplication and division. The second part required that the

student mentally round numbers and then mentally compute with those rounded numbers.

The interview protocol was designed by the researcher. The original questions were piloted with three children. The children were encouraged to ask for clarification of any aspect of the questioning procedure that did not seem clear and to offer suggestions. As a result, modifications were made with respect to what questions were asked and how they were phrased.

Questioning procedures were similar to those used by Cathcart (1969) in that the order and phrasing of the questions was preset to ensure consistency. The intention was to draw out the students' thoughts while they were estimating without prompting or suggesting possible answers.

The exercises selected were ones which are commonly encountered by grade six students in the regular course of mathematics instruction as prescribed by the Alberta Department of Education (1977).

Curriculum objectives for the sixth grade include addition and subtraction of any whole numbers; multiplication using one, two and three digit multipliers; division using one, two and three digit divisors (with or without remainders); mental multiplication of whole numbers by 10,100 and 1000; and estimation of products and quotients.

The addition exercises were all presented in column format of three, four or five numbers. The numbers had two, three or four digits.

The subtraction exercises involved three and four digit numbers with regrouping of the ones, tens and hundreds digits.

The multiplication exercises were either a one digit number times a two digit number or a two digit number times a three digit number. All exercises required regrouping.

The division exercises involved two digit divisors and either two or four digit dividends. Two quotients had remainders.

Experiences similar to all the exercises used can be found in the grade six textbooks of the three prescribed references. The publishers of these references are Addison Wesley; Holt, Rinehart and Winston; and Science Research Associates.

Part one consisted of two exercises each, of addition, subtraction, multiplication and division, in that order. For each part the simpler exercise was given first. The students were asked to explain how they would estimate the answer for each exercise.

Part two consisted of one exercise each of addition, subtraction, multiplication and division, in that order. The students were asked to round the numbers in each exercise and then compute the rounded numbers.

Each exercise in both parts was presented on a single page of a booklet which contained nothing but the exercises. After the student had made each estimate in part one, a pencil was used to do the exercise. The exercises in part two were not done with pencil and paper.

Exercises in Part one were presented in this form:

Addition: A1) $\begin{array}{r} 36 \\ 25 \\ 10 \\ +\underline{13} \end{array}$ A2) $\begin{array}{r} 203 \\ 169 \\ 46 \\ 77 \\ +\underline{\ \ 2} \end{array}$

Subtraction: S1) $\begin{array}{r} 743 \\ \underline{-416} \end{array}$ S2) $\begin{array}{r} 5862 \\ \underline{-377} \end{array}$

Multiplication: M1) $\begin{array}{r} 67 \\ \underline{X4} \end{array}$ M2) $\begin{array}{r} 253 \\ \underline{X39} \end{array}$

Division: D1) $16\overline{)92}$ D2) $76\overline{)6308}$

Exercises in Part two were presented in this form:

Addition: $\begin{array}{r} 2875 \\ 3629 \\ +\underline{\ 536} \end{array}$

Subtraction: $\begin{array}{r} 426 \\ -\underline{288} \end{array}$

Multiplication: $\begin{array}{r} 563 \\ \underline{X29} \end{array}$

Division: $84\overline{)3652}$

Data Collection

The data was collected over a one-week period from April 3 to April 7, 1978. Interviews were tape-recorded to facilitate subsequent analysis.

Pre-interview Discussion: A pre-interview discussion was conducted with each child to:

1) establish rapport and make the child feel comfortable,
2) ensure that the child understood what estimation means and why it is necessary,
3) explain the purpose of the study.

This part of the interview was not tape-recorded. The following statements and questions are examples of those posed by the researcher.

1) "The reason I am here is to find out how grade six students estimate answers to addition, subtraction, multiplication, and division exercises".
2) "The questions I will be asking you do not have right or wrong answers. I am interested in how you estimate the answers".
3) "Do you know what estimation means?" This question was followed by a discussion of the child's notion of estimation.

4) "When you make an estimate, you figure out in your head about how much the answer is without actually adding, subtracting, multiplying or dividing the numbers to get the exact answer as you would with a pencil and paper".

5) "Have you studied how to estimate in class?"

6) "Do you know why it is useful to be able to estimate answers?" This question was followed by a discussion of using estimation when shopping, building, solving problems or using the calculator.

7) Each student was asked not to discuss with the other students what was involved in the interview until all the students had participated.

<u>Part One</u>: The researcher gave each student the following directions before recording the answers:

1) "I am going to give you some exercises in addition, subtraction, multiplication and division. For each exercise I want you to explain, step by step, how you would estimate the answer in your head before you work out the exact answer with pencil and paper".

2) "I want you to try to tell me exactly what you think or say to yourself while you are making the estimate. It might help if you tell me how you would explain to, or teach another student how to estimate the answer".

3) "Do you understand what I am going to ask you to do? Do you have any questions before we begin?"

For each of the eight exercises to be estimated, some or all of the following questions were asked. Alternate questions were used when it was necessary to repeat the question. They were pre-planned to help the researcher be consistent and avoid asking leading questions when the students failed to respond.

1) "Please explain, step by step, how you would estimate the answer to this exercise".

Alternate questions were:

1a) "What would you think or say to yourself when estimating the answer to this exercise?"

1b) "What do you think is a good estimate of the answer to this exercise? Why? How did you get this answer?"

1c) "About how much do you think the answer is? Why?"

1d) "How would you explain to another student how to estimate this answer?"

2) "Do you think the exact answer is more or less than your estimate? Why?"

An alternate question was:

2a) "If you work out the exact answer with pencil and paper, do you think the exact answer will be bigger or smaller than your estimate? Why?"

3) "Now find the exact answer with this pencil. Do you think your estimate was a good one? Why or why not?"

An alternate question was:

3a) "Use this pencil to get the exact answer. Now that you know the exact answer, do you think your estimate was good? Why or why not?"

4) "If your estimate is not good, do you think there is a better way to estimate the answer to this exercise? Tell me how you would do it".

Part Two: The researcher gave each student the following directions before recording the answers.

1) "One way of estimating the answers to exercises is to round each number and then add, subtract, multiply or divide them".

2) "This time I'm going to ask you to do each exercise by rounding the number first".

For each of the four exercises to be estimated, some or all of the following questions were asked. Alternate ways of phrasing the questions were pre-planned to help ensure consistency.

1) "Please explain to me how you would round each of these numbers to make it easier to add (subtract, multiply, divide) them".

An alternate question was:

1a) "In order to make an estimate it is easier if you round the number first. How should you round these numbers to make it easier to get an answer?"

2) "Now, add (subtract, multiply, divide) the rounded numbers".

An alternate question was:

2a) "What answer do you get when you add (subtract, multiply, divide) using the rounded numbers?"

3) "Do you think this estimate is larger or smaller than the exact answer? Why?"

If the student was unable to respond to the first three questions, the following was asked:

4) "Do you know how to round numbers? Please explain how you would do it. Could you give me an example?"

Assumptions and Limitations

The following assumptions were made about the sample and procedure used:

1) It was assumed that verbal interaction with students while they estimated would provide information about what they were thinking.

2) It was assumed that grade six students would have an understanding of the operations used and the standard algorithms commonly applied.

3) The exercises were presented in a fixed order. It was assumed that no learning would affect subsequent responses.

Limitations of the study are acknowledged as follows:

1) The students' previous learning and background experiences were not taken into account.

2) Variation in the students' verbal ability may have affected the results.

3) The sample was limited in size.

4) Recent I.Q. scores or recent general mathematical ability scores were not available.

5) Presence of the tape recorder may have interfered with the responses.

Data Analysis Plan

The data were organized for analysis as follows:

Part One:

Accuracy of estimates: The estimates were classified as either accurate or inaccurate. The rationale for this distinction is given in Chapter IV. The data were examined to gain a general picture of the accuracy of the estimates both within and across the operations.

Strategies used: The strategies were classified on a researcher-designed basis. Major anticipated categories were: 1) use of rounding and 2) mental use of pencil and paper methods. The data were examined to determine if there was a relationship between the strategy used and the accuracy of the estimates.

Part Two:

Rounding and Computation: The rounding procedures were examined to determine how precisely, with respect to place value, the students rounded the numbers. The data were examined to determine if accuracy in computation is related to precision of rounding.

Accuracy and rounding: A comparison was made to see if there was a relationship between the ability to round numbers and use of this procedure to obtain accurate estimates in part one.

CHAPTER IV

DATA ANALYSIS AND DISCUSSION

Twenty-two grade six pupils were interviewed to determine what strategies they used as they estimated answers to addition, subtraction, multiplication and division exercises using whole numbers. The major purposes of the study were to determine:

(i) how accurately children estimate answers to computational exercises in addition, subtraction, multiplication and division;

(ii) identifiable strategies the children used when estimating;

(iii) if there were similarities in the strategies used to obtain "accurate" estimates;

(iv) if there were similarities in the strategies used to obtain "inaccurate" estimates;

(v) if there was a relationship between the ability to make accurate estimates and the ability to round numbers and compute with those rounded numbers.

The results of the study are presented and discussed in the present chapter under the following headings:

Accuracy of Estimates

Estimating Strategies

Rounding and Mental Computation

Accuracy of Estimates

To obtain a general picture of the accuracy of the estimates, each estimate was placed in a category based on the error, e (difference between the estimate and the exact answer).

N.A.E.P. (1976) noted that one is often concerned with estimating an answer of the proper order of magnitude. For example, if the largest place value of the exact answer is the thousands, hundreds, tens or ones place, the estimate should be within one thousand, one hundred, ten or one of the exact answer, respectively.

This suggested a rationale for the determination of accuracy categories. Estimates within the proper order of magnitude were placed in category A. Estimates which fell outside the proper order of magnitude were placed in category B. Category C accounted for the exercises for which no estimate was given. Table 2 shows the specific criteria for categories A and B.

Table 2

Criteria for determining categories to classify estimates on the basis of accuracy where e is the difference between the estimate and exact answer

Number of digits in exact answer	Accuracy Category A	Accuracy Category B
1	$e \leq 1$	$e > 1$
2	$e \leq 10$	$e > 10$
3	$e \leq 100$	$e > 100$
4	$e \leq 1000$	$e > 1000$

For purposes of further analysis and discussion, the estimates in category A were considered to be accurate; those in category B were considered to be inaccurate.

Table 3 gives the frequencies of the estimates in each accuracy category for each exercise.

Table 3

Frequency distribution of estimates in each accuracy category for each exercise (N=22)

Exercise	Accuracy Category			Total
	A	B	C	
A1	17	5	0	22
A2	17	5	0	22
S1	19	3	0	22
S2	17	4	1	22
M1	11	11	0	22
M2	0	21	1	22
D1	6	16	0	22
D2	0	21	1	22
Total	87	86	3	176

The data in Table 3 show that about half the estimates were considered accurate and about half were considered inaccurate.

By combining the number of accurate estimates for each operation it can be seen that generally, the most accurate estimates were made for the operations in this order: subtraction, addition, multiplication, and division.

The addition exercises were both relatively well estimated. Only five estimates in each addition exercise were considered inaccurate.

The subtraction exercises were both relatively well estimated. All but three estimates made for S1 were considered accurate. This was the best estimated exercise. All but five estimates for S2 were considered accurate.

The multiplication exercises were relatively poorly estimated. Only half of M1 and none of the M2 estimates were considered to be accurate.

The division exercises were relatively poorly estimated. Only about one-quarter of the D1 and none of the D2 estimates were considered accurate.

Because addition is usually considered easier than subtraction, one would have expected the addition estimates to be more accurate than the subtraction estimates. While it would be difficult to establish the relative difficulty of the specific exercises used, it may be that A1 and A2, which required combining four and five numbers respectively, were more difficult than S1 and S2.

Table 4 shows the frequency of estimates which were the same as, above, or below the exact answer in each category for each exercise.

Table 4

Frequency distribution of estimates the same as, above or below the exact answer in each accuracy category for each exercise

	Accuracy Category					
	A			B		C
Exercise	Exact	Low	High	Low	High	
A1	4	6	7	3	2	0
A2	2	10	5	5	0	0
S1	3	8	8	1	2	0
S2	1	6	10	4	0	1
M1	4	4	3	8	3	0
M2	0	0	0	19	2	1
D1	0	5	1	10	6	0
D2	0	0	0	16	5	1
Total	14	39	34	66	20	3

The data in Table 4 show that for the accurate estimates there was about equal tendency to be above the exact answer or below the exact answer. For the inaccurate estimates, however, about three-quarters of the estimates were below the exact answer, and only one-quarter were above the exact answer. This indicates that perhaps the more uncertain the student was about making an estimate, the more likely he was to underestimate.

After estimating the answer to each exercise, the students were asked to check their estimate by doing the exercise with pencil and paper. The errors made were generally classified as either algorithmic or non-algorithmic. Algorithmic errors were the result of using the algorithmic process incorrectly. Students who made non-algorithmic errors used the correct algorithmic process, but made casual or careless errors.

In the division exercises, some students just multiplied their estimate for the quotient by the divisor rather than doing the long division. All students who chose to do this multiplied correctly. Table 5 shows the frequency of correct and incorrect answers for each exercise.

Table 5

Frequency distribution of correct and incorrect answers to each exercise done with pencil and paper

	Answer Type					
Exercise	Correct	Algorithmic error	Non-algorithmic error	Unable to do	Checked division by multiplication	Total
A1	18	0	4	0	0	22
A2	18	0	4	0	0	22
S1	21	1	0	0	0	22
S2	18	1	3	0	0	22
M1	21	1	0	0	0	22
M2	11	2	8	1	0	22
D1	9	5	3	2	3	22
D2	7	5	0	3	7	22
Total	123	15	22	6	10	176

When doing the exercises with pencil and paper, the most errors were made for the operations in the following descending order: division, multiplication, addition and subtraction. This order is consistent with the order of accuracy of the estimates across the operations. In the addition exercises, only four students made errors in A1 and four students made errors in A2. These were all non-algorithmic errors and only one student did both exercises incorrectly. The number and nature of the errors indicates that the students were generally

skillful at column addition.

In the subtraction exercises, only one student made an error in S1 and four students made errors in S2. The only student who did both exercises incorrectly made algorithmic errors. The other three errors were non-algorithmic. The number and nature of these errors indicates that the students were generally skillful at subtraction.

In the multiplication exercises, only one student did M1 incorrectly and that was an algorithmic error. Only half of the students did M2 correctly. Of the eleven students who did not, eight made non-algorithmic errors, two made algorithmic errors, and one student was unable to do the multiplication. The number of errors indicates that the students could multiply simple (one digit times two digits) exercises well, but had difficulty with harder (two digit times three digits) exercises. As most errors were non-algorithmic, the difficulty appears to lie in carelessness or lack of practice rather than faulty understanding of the algorithm.

In D1, three students checked their estimates by multiplying correctly. Nine did the division algorithm correctly and ten (almost half) did it incorrectly. Three made non-algorithmic errors and five made algorithmic errors. Two students were unable to do D1.

In D2, seven students checked their estimates by multiplying correctly. Seven students did the algorithm correctly and five did it incorrectly. All five made algorithmic errors. Three students were unable to do D2.

The number and nature of the errors in the division exercises indicated that generally, the students had difficulty with division.

Generally, both the estimation and pencil and paper calculation of the addition and subtraction exercises were well done. Also, both the estimation and pencil and paper calculation of the multiplication and division exercises were not well done. This parallel observation was not surprising as addition and subtraction are generally regarded as easier operations than multiplication and division.

Estimating Strategies

The identifiable strategies used by the students when estimating answers were generally classified as follows:

1) Student attempted to calculate the exact answer using standard paper and pencil techniques mentally. (Method M)

2) Student attempted to apply some rounding procedure to simplify the calculation. (Method R)

Responses from students who did not use identifiable strategies were classified as follows:

1) Student could not clearly express the procedure used (Response U).

2) Student offered no explanation of the procedure used. (Response N).

It could be argued that method M is not a true estimation procedure because it is an attempt to obtain the exact answer. This method focuses on detail and step by step algorithmic procedures. Use of this method may be an indication that the student does not distinguish between the meaning of an operation and a standard algorithm.

Method R, on the other hand, may be viewed as a flexible wholistic approach focusing on the meaning of the operation. It is interesting to note that the standard "round and calculate" strategy taught as an estimation technique can be used as mechanically as method M.

Table 6 shows the frequency of each response or method corresponding to the estimates given for each accuracy category. Since twenty-two students each were presented with eight exercises, the total possible number of estimates is 176. However, one student could not even attempt to estimate exercise D2, so the total number of responses represented in Table 6 is 175.

Table 6

Frequency of each response or method corresponding to the estimates given for each accuracy category

	Response or Method				
Accuracy category	M	R	U	N	Total
A (accurate)	42	34	9	2	87
B (inaccurate)	20	29	17	20	86
C (no estimate)	2	0	0	0	2
Total	64	63	26	22	175

The data in Table 6 reveals several facts about the responses and use of the strategies. Methods M and R were used equally as often, each method accounting for about one-third of all estimates made. All responses and methods were associated with both accurate and inaccurate estimates. About one-third of the method M estimates and about one-half of the method R estimates were inaccurate. Two-thirds of the estimates associated with response U were inaccurate and about nine-tenths of the responses associated with response N were inaccurate. These facts indicate that the more accurate estimates were not the result of a particular general strategy, but perhaps that when a student is aware of applying a particular strategy, the estimates were more accurate.

Table 7 shows the frequency of each response or method associated with the estimates for each operation. Since twenty-two students were each presented with two exercises for each operation, the total possible estimates for each operation is 44. However, one student could not even attempt to estimate exercise D2, so the total estimates represented for division is 43.

Table 7

Frequency of each response or method associated with estimates for each operation (N=22)

Operation	Response or Method				
	M	R	U	N	Total
Addition	19	21	4	0	44
Subtraction	24	14	5	1	44
Multiplication	17	16	7	4	44
Division	4	12	10	17	43
Total	64	63	26	22	175

The data in Table 7 show that the different responses or methods associated with the estimates cut across the operations, but responses U and N are most frequently associated with multiplication and division.

In order to draw conclusions about successful strategies and problems which the students had with estimation, the particular way in which the methods were used must be examined.

Method M

1) Addition: Method M in addition means an attempt to mentally add the numbers by columns.

Seven students used method M to obtain accurate estimates for exercise A1. Six of these seven used the standard algorithm of right to left column addition, with four obtaining the exact answer. The seventh student added the numbers from left to right.

Six students used method M to obtain accurate estimates for exercise A2. Four of these six used the standard algorithm, with three obtaining the exact answer. Two students added the columns from left to right.

Four students who used method M for exercise A1 obtained inaccurate estimates. Of the three students who used the standard algorithm, one obtained the exact answer of 84 and then rounded it to 50. The fourth student explained how to use the standard algorithm and then made a guess of 21.

Two students who used method M for exercise A2 obtained inaccurate estimates. Both students used the standard algorithm.

Basically, success in using method M for addition depends on skill and accuracy in remembering partial sums and regrouped numbers which are normally written down, and a sound knowledge of basic facts.

2) Subtraction: Method M in subtraction means an attempt to mentally subtract the numbers by columns.

Ten students used method M to obtain accurate estimates for exercise S1. Six of these ten used the standard decomposition algorithm, with three obtaining the exact answer; one student obtained the exact answer and then rounded it. Four students subtracted the numbers from left to right.

Eleven students used method M to obtain accurate estimates for exercise S2. Eight of these eleven used the standard algorithm, with one obtaining the exact answer. Three students subtracted from left to right.

One student who used method M for exercise S1 obtained an inaccurate estimate. This student used a left to right method of column subtraction, subtracting the smaller digit from the larger in each case. He forgot one of his partial answers and gave 733 as his

estimate instead of 333.

Two students who used method M for exercise S2 obtained inaccurate estimates. One student attempted to use the standard algorithm, but could not do it. The other student subtracted from left to right and confused the columns.

Success in using method M for subtraction seems to be based on the ability to remember partial difference and "decomposed" numerals which are normally written down, as well as a sound knowledge of basic facts. As the number of digits in the numerals and the need to regroup increases, mental use of the standard algorithm to obtain the exact answer becomes more difficult. An interesting way of obtaining an accurate estimate was to subtract the columns one at a time, from left to right, subtracting the smaller from the larger digit in each case.

3) Multiplication: Method M in multiplication means an attempt to mentally multiply the numbers using the standard algorithm.

Six students used method M to obtain accurate estimates for exercise M1. These students all used the standard algorithm with four obtaining the exact answer.

No students used method M to obtain accurate estimates for exercise M2.

Five students who used method M for exercise M1 obtained inaccurate estimates. These students attempted to use the standard algorithm , but were unable to remember all the partial products.

Five students who attempted to use method M for exercise M2, obtained inaccurate estimates. Another student who attempted to use method M was unable to give a final estimate.

This method for multiplication is difficult because partial products and carried numbers must be remembered. While some students had success with an easy example (one digit times two digits), none could use this method successfully with the more difficult example (two digits times three digits).

4) Division: Method M in division means an attempt to mentally use the inverse process of multiplication or the standard long division, to obtain the exact answer.

Two students used method M for exercise D1. They mentally multiplied the divisor by trial numbers to obtain a product close to the dividend. Both students obtained accurate estimates.

One student used method M for exercise D2. This student attempted to use the standard division algorithm and obtained an inaccurate estimate.

The standard long division algorithm is difficult to use either mentally or with pencil and paper because it involves several operations and steps. This may explain why so few students attempted to use this method when estimating.

Generally, method M is difficult because numbers and partial answers, which are usually recorded, must be remembered. Use of this method becomes more difficult as the numbers increase and the number of steps increase.

Of the students who used method M and were not satisfied with their estimates after doing the examples with pencil and paper, few could suggest alternate methods.

Method R.

1. Addition:

Seven students used rounding to obtain accurate estimates for exercise A1. Four of these seven rounded each number to the nearer ten and added the rounded numbers, with one adjusting the answer. One student rounded two numbers to the nearer five, two to the nearer ten, added the rounded numbers and adjusted the sum. One student rounded the sum of the first two numbers and added the last two. The seventh student approximated the two partial sums of the numbers taken in pairs, and adjusted the sum of these.

Ten students used rounding to obtain accurate estimates for exercise A2. The techniques included rounding each number (to various places) and adding, rounding partial sums and adding, and a combination of these techniques, with some adjusting answers.

One student used rounding and obtained an inaccurate estimate for exercise A1. The student rounded three numbers to the nearer ten, one to the nearer five, added these to get an answer of 95 and then again rounded up to get 110.

Three students used rounding and obtained inaccurate estimates for exercise A2. One student rounded numbers to the nearer ten, but added incorrectly. One estimated two partial sums, but added them

incorrectly. The third student estimated by considering the hundreds digits only.

Those who successfully used method R for addition did not necessarily follow a rigid pattern, but used creative and flexible strategies. All numbers were not necessarily rounded to the same place value. Partial sums of a group of two or three numbers were used as subestimates which were combined. Adjustments were made in some cases for numbers which were rounded up or down.

Inaccurate estimates seemed to result from inaccurate mental computation and not poor or uncertain strategy.

2. Subtraction:

Seven students used rounding to obtain accurate estimates for exercise S1. Four of these seven rounded each number to the nearer hundred and subtracted. Two students rounded the numbers to the nearer ten or five and subtracted incorrectly, but the error was small. One student estimated that the subtrahend was about half of the minuend and took approximately half of the minuend for an estimate.

Five students used rounding to obtain accurate estimates for exercise S2. One student rounded both numbers to the nearer hundred and subtracted. Two students rounded one number to the nearer thousand, the other to the nearer hundred, and subtracted. One student claimed to use rounding but did not explain how it was used. The fifth student rounded numbers, but subtracted incorrectly although the error was small.

One student used rounding and obtained an inaccurate estimate for exercise S1. The student rounded each number to the nearer hundred but subtracted incorrectly.

One student used rounding and obtained an inaccurate estimate for exercise S2. The student rounded the thousands as though they were hundreds.

Those who used method R most successfully for subtraction rounded both numbers to one significant digit or both to the same place value. Of the two students who used method R to obtain inaccurate estimates, one made a rounding error and the other made a computational error.

3. Multiplication:

Four students used rounding to obtain accurate estimates for exercise M1. Three of these four rounded correctly to one significant figure, two multiplying correctly and one incorrectly. The fourth student rounded incorrectly to one significant figure.

No students used method R to obtain accurate estimates for exercise M2.

Four students used method R and obtained inaccurate estimates for exercise M1. One student rounded incorrectly to one significant figure and multiplied incorrectly. One student did not round appropriately and multiplied incorrectly. Two students rounded one number to zero to obtain a product of zero. One student realized this was not appropriate and made a guess of 100 for an estimate.

Eight students used rounding and obtained inaccurate estimates for exercise M2. Five of these eight rounded both numbers to the nearer ten but multiplied incorrectly. The other three both rounded inappropriately and multiplied incorrectly.

One reason for difficulty with method R for multiplication is failure to round appropriately. This means rounding the number to too many significant figures for convenient mental multiplication. The most frequent error, however, was mental multiplication of multiples of ten.

4. Division:

One student used rounding to obtain an accurate estimate for exercise D1. The student rounded both numbers to the nearer ten, divided and adjusted the answer.

No students used rounding to obtain accurate estimates for exercise D2.

Four students used rounding and obtained inaccurate estimates for exercise D1. The students rounded correctly, three rounding both numbers to the nearer ten, but all divided incorrectly.

Seven students used rounding and obtained inaccurate estimates for exercise D2. The students rounded the numbers correctly, but divided incorrectly.

Use of method R in division depends on both appropriate rounding and mental division of rounded numbers.

Generally, use of method R requires several sub-skills, some of which are hard to define. It requires knowledge not only of rules

for rounding numbers, but also a feel for rounding them to an appropriate number of significant figures suitable to the particular exercise and operation. This method also requires skill in mental computation of rounded numbers for all operations.

Rounding Numbers and Mental Computation

Part two of the interview involved one exercise each in addition, subtraction, multiplication and division. The student was asked to round the numbers in each exercise to make them easier to compute, and then compute them mentally. Table 8 shows the frequency of correct and incorrect mental computation for each operation with respect to precision of rounded numbers. One student was unable to attempt to round numbers, so the total number of estimates for each operation is 21.

The majority of the children rounded all numbers to one significant figure; two-thirds did for the addition, subtraction, and multiplication exercises, and half did for the division exercises.

Table 8

Frequency of correct and incorrect mental computation for each operation with respect to precision of rounding (N=21)

Mental Computation		Precision of rounding numbers: One significant figure	Precision of rounding numbers: More than one significant figure
Addition	correct	12	0
	incorrect	3	6
Subtraction	correct	13	1
	incorrect	2	5
Multi-plication	correct	3	0
	incorrect	12	6
Division	correct	0	0
	incorrect	11	10

With one exception, in subtraction, none of the students rounding to more than one significant figure were able to compute the rounded numbers correctly.

Of the twenty-two students, twelve were able to both correctly round and correctly add mentally. Of these twelve, five used rounding in both exercise A1 and exercise A2, two used rounding in exercise A2 only, and five did not use rounding in either exercise A1 or A2.

Fourteen students were able to both correctly round and correctly subtract mentally. Of these fourteen, five used rounding in both exercise S1 and exercise S2, one used rounding exercise S1 only, and eight did not use rounding in either exercise S1 or S2.

Only three students were able to both correctly round and correctly multiply mentally. Of these three, one used rounding in both exercise M1 and exercise M2, one used rounding in exercise M2 only, and one did not use rounding in either exercise M1 or exercise M2.

No students were able to both correctly round and correctly divide mentally.

The students who could both correctly round numbers and correctly compute mentally did not necessarily use these skills in part one.

Students who rounded numbers to more than one significant figure had difficulty computing those rounded numbers. Those who rounded numbers to one significant figure had difficulty with the number of zeros when computing.

An inconsistency noted was that some students who did not both correctly round and correctly compute exercises in part two, used these techniques to obtain accurate estimates in part one in the same operation. This observation accounted for seven accurate addition estimates, three accurate subtraction estimates, and three accurate multiplication estimates.

CHAPTER V

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

Summary

Mathematics educators agree that estimation is an important and necessary skill for modern living. In spite of its significance, estimation has been relatively neglected in the areas of both teaching and research.

The major purposes of this study were to determine:

(i) how accurately children estimate answers to computational exercises in addition, subtraction, multiplication and division;

(ii) identifiable strategies the children used when estimating;

(iii) if there were similarities in the strategies used to obtain "accurate" estimates;

(iv) if there were similarities in the strategies used to obtain "inaccurate" estimates; and

(v) if there was a relationship between the ability to make accurate estimates and the ability to round numbers and compute with those rounded numbers.

A two-part interview was conducted with each student on an individual basis. The first part required that the student orally estimate answers to computational exercises in addition, subtraction, multiplication and division. The second part required that the student mentally round numbers and then compute with those rounded numbers. The interviews were tape-recorded to facilitate subsequent analysis.

Conclusions

Keeping the limitations of the study in mind, the following conclusions have been suggested by the results.

1. About half the estimates were classified as being accurate and about half were classified as being inaccurate. The students were more accurate in estimating those operations which they could do best when using pencil and paper. When unsure of estimating, the students tended to underestimate rather than overestimate.

2. The majority of the strategies used could be generally classified as either using rounding procedures or using pencil and paper algorithms mentally. The latter procedure may have been prompted by the manner in which the exercises were presented.

3. The use of a particular strategy was not associated with the more accurate estimates.

4. The use of a particular strategy was not associated with the less accurate estimates.

5. The ability to round numbers and mentally compute with those rounded numbers was neither a necessary nor a sufficient condition for making accurate estimates. This process can be used mechanically, but use of context in estimation exercises may lead to a more rational application.

Recommendations for Teaching and Research

The present study was limited to exercises which were presented in written form and the estimation was done without pencil and paper. These exercises were not presented in a situational or a practical context, but involved numerical computation only.

The results of this study suggest that the foregoing criteria limited and perhaps directed the students' responses. The following recommendations are intended to suggest how teachers may translate the findings into teaching practices and how further research in this area might overcome some of these limitations.

Recommendations for Teaching

Results of the study have the following implications for the teaching of estimation:

1) The reason for making an estimate must be used as a guide to select a strategy and to judge the reasonableness of an estimate. Rounding must not be taught as a mechanical process where the method justifies the answer. The possibility of this happening is suggested by the students who accepted estimates of zero.

2) In practicing mental computation with rounded numbers, zeros must be given regular attention. The necessity for this was shown by the students who rounded correctly and knew their basic facts but could not compute accurately when zeros were involved.

3) To evaluate a student's ability to estimate, it is necessary to consider the context of the problem, the reason for making the estimate and the appropriateness of the accuracy chosen.

4) Teaching the skills of rounding numbers and computing mentally with rounded numbers does not necessarily mean that these skills will be transferred to the process of estimation. Teachers must be careful to ensure that these skills are not viewed as an independent procedure. The need for this caution was shown by the students who could both correctly round numbers and compute with them but did not use these skills in part one estimates.

5) There is a need to separate the concepts or meanings of operations from the standard algorithms which are applied. Students who used method M to estimate answers may not have made this distinction.

6) Students need considerable practice in applying estimation skills. This was demonstrated by the students who were inconsistent in using rounding and computation skills correctly in both parts of the study.

Recommendations for Research

Results of the study had the following implications for further research on estimation:

1) The form in which the exercises were presented to the students may have prompted the mental use of paper and pencil algorithms. Computational exercises should be presented in a variety of forms such as standard vertical, standard horizontal, arithmetic

sentence, or oral, as the form may influence the choice of strategy used. In particular, the mental use of pencil and paper algorithms would be awkward if the problems were presented orally. When attempting to gain insight into a student's thinking it is important to reduce the effect of any factors which may suggest answers or act as cues.

2) Estimation of computation should be presented in a variety of contexts, as the context often suggests the suitable precision for rounding numbers. In different practical situations, desired accuracy may vary even though the numbers and operations involved are identical.

3) The order of accuracy of estimates across operations differs from the order which Hall (1947) found in the ability to solve verbal problems without pencil and paper. Because true estimation techniques differ from pencil and paper algorithms, it would be useful to investigate which, if any, strategies should vary depending on the complexity of a particular exercise within any operation, or vary for different operations. Results of such a study would have implications for the teaching of estimation. For example, rounding strategies for simple multiplication exercises may differ from those used for more difficult exercises, and appropriate rounding strategies for division may differ from those appropriate for subtraction.

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